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**MANPOWER ALLOCATION AND MATHEMATICAL PROGRAMMING**

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## MANPOWER ALLOCATION AND MATHEMATICAL PROGRAMMING

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1. Manpower allocation problems are a special instance of resource allocation problems--themselves specializations of general decision problems. In defining a decision problem, one specifies the available courses of action, the consequences of these courses of action, the objectives sought in dealing with the problem, and the consideration to be given to random effects. Dealing with a decision problem generally involves four steps:

- (1) specifying a model--a set of assumed empirical relationships among variables,
- (2) stating a subset of decision variables--those variables the decision-maker is free to control,
- (3) specifying an objective function, or criterion, which is affected by the values of the decision variables, and
- (4) stating a procedure for analyzing the effect on the objective function of alternative values of the decision variables.

A very significant portion of the general decision problem consists of specifying a suitable model of the process--one, which in Karlin's words, "...abstracts from reality in a way which preserves the essential structure of the problem in such a way that its analysis affords insight into both the original concrete situation, and other situations which

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have the same formal structure."<sup>2</sup> Mathematical programming is simply the mathematics of maximizing a function of several variables which are subject to constraints. The field of constrained optimization is broader and includes a variety of search procedures<sup>3</sup> and gradient methods.<sup>4</sup> Mathematical programming provides a convenient and natural conceptual framework with which to view many decision problems, and can yield as a computational by-product a much greater variety of subsidiary information than other optimization techniques or purely descriptive models.

There have been a few references to the use of mathematical programming in manpower allocation,<sup>5,6,7,8</sup> but no systematic attempt to use available tools is apparent. The failure to exploit mathematical programming in manpower allocation problems is understandable given the general lack of agreement on personnel cost or value measures, the relative paucity of knowledge concerning empirical relationships between policy variables (such as re-enlistment bonuses) and resulting actions (such as re-enlistments), and the only gradually increasing availability of data relevant to manpower decisions. When problems have been cast in this framework it has been either to exhibit the structure of a solution, or to perform a specific study applicable to one specific set of data. The advent of large-capacity computer systems with remote

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<sup>2</sup> Karlin, S., Mathematical Methods and Theory in Games, Programming, and Economics, Vol. I, Addison-Wesley, Reading, Mass., 1959.

<sup>3</sup> Wilde, D. J., Optimum Seeking Methods, Prentice-Hall, Englewood Cliffs, New Jersey, 1964.

<sup>4</sup> Arrow, K. J., E. Hurwicz, and H. Uzawa, Studies in Linear and Non-Linear Programming, Stanford, California, Stanford University Press, 1958.

<sup>5</sup> Kossack, C. F., and R. E. Beckwith, The Mathematics of Personnel Utilization Models, WADC-TR-59-359, Lackland AFB, Texas, November 1959.

<sup>6</sup> Gorham, W., An Application of a Network Flow Model to Personnel Planning, RM-2587, The RAND Corporation, June 1960.

<sup>7</sup> Kagen, H., Unpublished Notes on Rotation Models, Randolph AFB, Texas, July 1966.

<sup>8</sup> Sorenson, R. C., "Logical Model Representing Flow in the U.S. Army: Considerations Relative to Reduction in Turbulence," Technical Research Note 156, U.S. Army Personnel Research Office, July 1965.

terminals and access to actual manpower and personnel data should increase the use of mathematical programming techniques by decision-makers, as well as analysts. The manpower planner should be able to query on-line optimization routines which operate on timely data to obtain initial schedules, courses of action, or allocation plans.

Aside from day to day use by the decision-maker, a great deal of value accrues from formulating and analyzing a problem as a mathematical programming problem. Retaining the same set of constraints, one may experiment with various objective functions, or various weightings of several objective functions to ascertain the sensitivity of solutions to criteria. Through parametric and postoptimality analyses one can determine how minor changes in data or coefficients affect a solution. And the dual variables--or shadow prices--provide valuable information on the importance of various constraining relationships.

Formulating a mathematical programming problem forces explicit definition of the constraints operating on the system, and the type of criteria one will consider. One must concentrate on the empirical relationships among the variables, and on the type of data required to perform the analysis. Often this calls into question existing policies and goals in an objective way not otherwise possible. Practically, computational economies can result from casting problems as mathematical programming problems because of the existence of computing algorithms and routines in most computing centers. Standard capacitated transportation algorithms and linear programming routines are almost always available, averting a good deal of programming effort. Many problems are of course non-linear, either in objective function or constraints, but in many instances the problem may be approximated by a linear programming formulation over the relevant range. Several such linear approximations and parametrizations of the coefficients and constraints can yield insight into the behavior and sensitivity of the solution, and are certainly more valuable than failure to perform the analysis because of the recognized non-linearities of the original problem.

Recognizing that a particular manpower allocation problem can be viewed as a mathematical problem only begins the task of analysis.

Important practical barriers remain. The foremost may be the conceptual definition of the technological or objective function coefficients. In considering the movement of Air Force personnel between grades, specialties, and skill levels, an obvious characterization is a Markov process, and the mathematical programming tool which comes to mind is the optimization of some cost or value criterion within the structure of the Markov process, while satisfying some set of requirement constraints. But what is the value or cost of a given person in a given state? Or what is the cost or value of having a person transition between two states? Subsistence, training, and budget costs enter, but so does the relative importance of having a particular capability within the Air Force. Even given the definition of the various coefficients, estimates of numerical values may still be difficult to obtain in the manpower and personnel area. It has only been in the last four or five years that personnel records have systematically been kept in a form usable by computer systems. Cost and productivity figures are inherently difficult to generate because of the existence of joint products, joint usage, variable productivity of human beings, and the many mixes of labor, capital, and organizational structures existing. The cost and effort involved in obtaining data may be considerable. Also, a realistic formulation of an actual problem may be prohibitively large for current computer systems. This consideration shaped the Rotation Base Model which will later be described.

In general, manpower allocation problems appear to involve a natural decomposition by specialty, or groups of specialties, and this may lead to computationally feasible problems. Even if problems are excessively large, a combination of heuristic and formal analysis can usually reduce the problem to tractable size. While the time and cost in personnel to formulate a model, obtain data, construct and check out the working model, and begin to use the results is not to be underestimated, one of the derivative benefits in working with mathematical programming models has always been the increase in knowledge and expertise of the analysis team. This makes it important that model design, construction, and use be coordinated closely with the agency which will ultimately use the model or solutions. The danger of an

analysis effort separated from the user's staff is the "Not Invented Here" syndrome.

2. In evaluating the effects on the Air Force of the recent OSD Civilianization Program, we became aware of the "rotation base requirement" problem. This requires specifying the number of military personnel in each grade, Air Force Specialty Code (AFSC), and skill level required in the United States (ConUS) to support overseas military personnel requirements. The ConUS base must be sufficiently large to allow personnel to serve reasonable long tours in the United States between tours in overseas areas. While methods were available to estimate this rotation base,<sup>9</sup> we thought it desirable to construct a more accurate estimating tool.

We initially simplified the problem by creating aggregates of AFSC's, grades, and skill levels--for example all Communications/Electronics personnel at the journeyman skill level, in the grades of Airman First Class and Staff Sergeant--and denoted these as an "assignment class." We then aggregated all geographic assignments of the same type and tour length as "tour areas." This type distinction permits differentiation between a twelve-month tour in, say, South Vietnam, and a twelve-month tour in Dakar. We assumed a static situation with requirements and personnel policies known and unchanging. The model was to determine a set of personnel rotation and training assignments which met requirements at all bases, and which used the minimum number of military personnel.

We conceive the rotation and assignment process as being a network of nodes and arcs. Each node is an assignment class in a particular tour area--for example, Communications/Electronics personnel in ConUS. Each arc connects two nodes and represents a possible assignment action. The basic requirement forcing personnel flow within the system is the requirement that personnel rotate out of all overseas bases at the end of their tours, and that all requirements be continuously met.

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<sup>9</sup>Kagen, op. cit.

In actually dealing with the problem of rotation and reassignment, the Air Force recognizes that some specialties are capable of being utilized immediately in other jobs, and that with some cross-training, new skills can be acquired. Furthermore, assignment classes which are surplus in ConUS can be cross-trained and rotated overseas with a shorter ConUS tour than assignment classes which are required in ConUS. We constructed the model to recognize this "surplus" labor pool, and to take advantage of cross-training opportunities wherever cross-training seemed to offer a manpower saving. Within the model an arc (assignment possibility) connects every pair of assignment classes between which personnel can move with cross-training. Training is not free in manpower terms--it ties up military personnel. For instance, if 100 men per month are entered into a two-month aircraft mechanic course, the total manpower pool must allow for the 200 men who are continuously in training.

It is the sum of all personnel in training and all personnel serving at all bases that the model seeks to minimize. This manpower expression is composed of three parts--personnel in training, personnel assigned to meet requirements at bases, and personnel excess to ConUS requirements but necessary in ConUS to support the rotation flow. If personnel are available at a ConUS node in excess of requirements at that node, the model allows those personnel to be entered into training pipelines for assignments to other AFSC's without requiring additional manpower to fill the pipelines. It is the "free" substitutability of some classes of personnel that make manpower economies possible within this model and within the Air Force.

The abstracted rotation problem is thus to minimize total manpower at all nodes,  $i$ , and over all arcs  $(i, j)$  subject to the requirement that personnel at overseas bases be returned at a rate dependent on the overseas tour length. This is a simple problem conceptually and we decided to include about 50 assignment classes and 20 tour areas--1000 nodes. Any attempt to leave this problem in the form of a standard linear programming problem rapidly generates about 2000 equations and possibly 3000 variables--well beyond the computational capability of standard linear programming routines. By using a bounded variable

algorithm, it might be possible to reduce the problem to 1000 constraints--still formidable. We therefore chose to handle the network as a network--although a complex and clumsy one--and used a capacitated transportation algorithm as the computational mechanism. The Fulkerson out-of-kilter algorithm<sup>10</sup> is an efficient method for finding a minimum cost circulation in a capacitated network, and it is the algorithm used. Due to the summation constraints on the system, and the necessity to distinguish between surplus personnel and required personnel, the network becomes quite complex, and the number of arcs proliferates furiously. However, we are able to deal with the typical 50 assignment class by 20 tour area problem quite easily and without excessive computer time. This is the final functional criterion by which we judge this first Rotation Base Requirements Model.

It might appear that having formulated the problem mathematically, and having decided to use the out-of-kilter algorithm, the task was finished. But to construct a fairly complex network by hand is a tedious procedure. As in most such modeling-programming efforts, the primary tasks were constructing logical routines to translate relevant inputs in standard form--lists of requirements by AFSC, by base, for instance--into the network description viewed by the out-of-kilter algorithm. A post processor, or set of report generators, was constructed to take the results of the out-of-kilter computations--expressed only as raw flows--and interpret these into assignment and rotation information meaningful to a manpower planner. The task of computer programming and checkout required four months of a skilled mathematician-programmer's efforts. The initial model has been briefed and distributed to various interested offices of the Air Force. The model is written entirely in Fortran IV, and as soon as it is fully documented, there should be no difficulty in making it available to other interested users.

During its construction and use we have become aware of other factors which affect the rotation base question, and we are constructing a new model, RBM-II, which will consider accession and loss

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<sup>10</sup> Fulkerson, D. R., An Out-of-Kilter Method for Minimal Cost Flow Problems, The RAND Corporation, Paper P-1825, Jan. 18, 1960.

rates, training school capacities, and training costs. These features will be grafted onto the already complex network, but we see no alternative way to treat a problem of this magnitude.

As an example of the use of the model, consider the following hypothetical situation. Suppose we have the following requirements in three assignment classes in two tour areas.

Table 1  
MANPOWER REQUIREMENTS

Assignment Class	Tour Area	
	ConUS	Overseas
Missile Maintenance	75,000	0
Communications/Electronics	15,000	8,000
Aircraft Maintenance	0	10,000

With no cross-training, given a 48-month ConUS tour, and a 12-month overseas tour, the total ConUS manpower requirement to support this rotational situation is 147,216 men. If we permit cross-training according to Table 2, we would intuitively expect to use the large supply personnel assignment class 1 (missile maintenance) to supply the requirements of assignment class 3 overseas.

Table 2  
CROSS-TRAINING TIMES (MONTHS)

From	To		
	Missile Maint.	Comm./Elec.	Aircraft Maint.
Missile Maint.	0	2	1
Comm./Elec.	4	0	1
Aircraft Maint.	1	2	0

The optimal rotation pattern calculated by the Rotation Base Model is shown in Fig. 1.

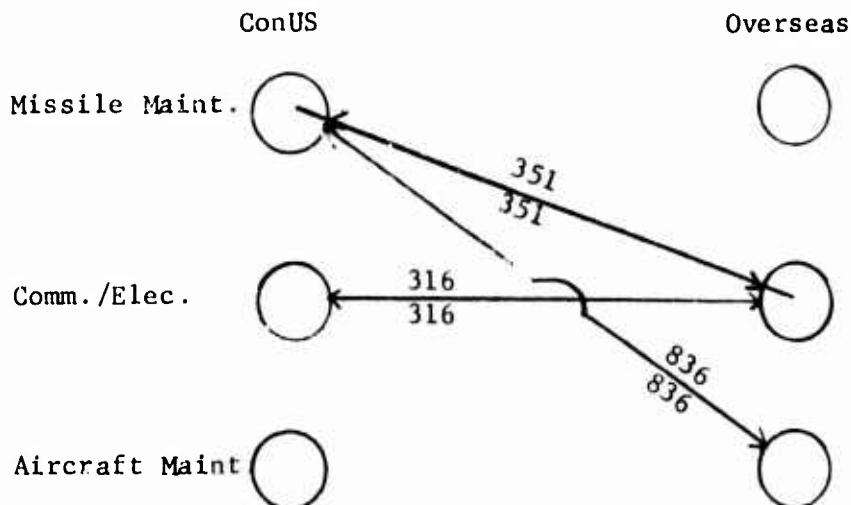


Fig. 1 -- Monthly Personnel Reassignments

In this rotation pattern there are 3514 personnel in training, 75,072 on duty at ConUS assignment class 1, and 15,168 on duty at ConUS assignment class 2, a total required rotation base of 93,754 compared to 147,216 in the original independent rotation base requirement.<sup>11</sup>

Assume that a policy decision is made to extend all tours for personnel in this overseas tour area from 12 months to 16 months. The new optimal flow pattern is shown in Fig. 2.

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<sup>11</sup> Requirements are not met exactly due to the truncation which occurs in the integer arithmetic of the model.

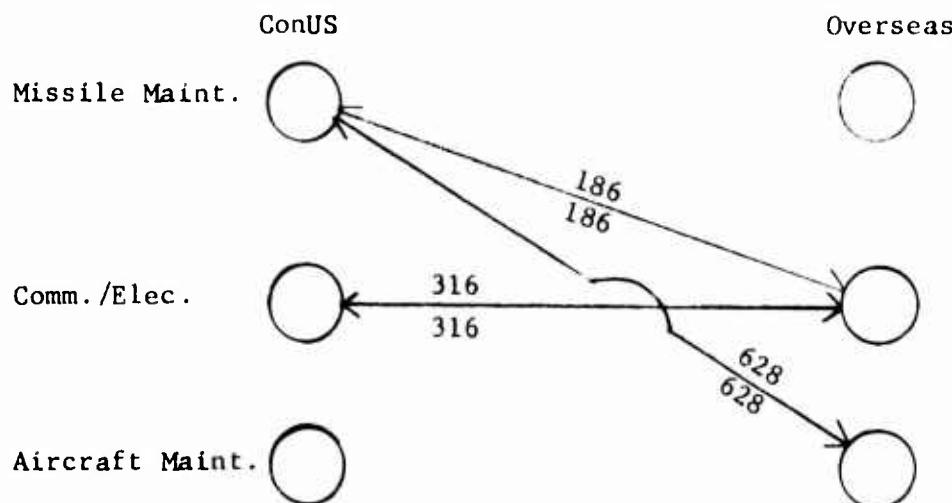


Fig. 2 -- Personnel Flow Pattern With 16-Month Overseas Tour

The total manpower in training and in the rotation base is 92,447, a reduction of 1307 due to the tour length extension.

Assume again that in the original hypothetical example one desired to know the total manpower saving generated by a reduction of 4000 in assignment class 3 overseas. The revised flow pattern is shown in Fig. 3 and indicates a saving of 620 personnel in the training pipeline.

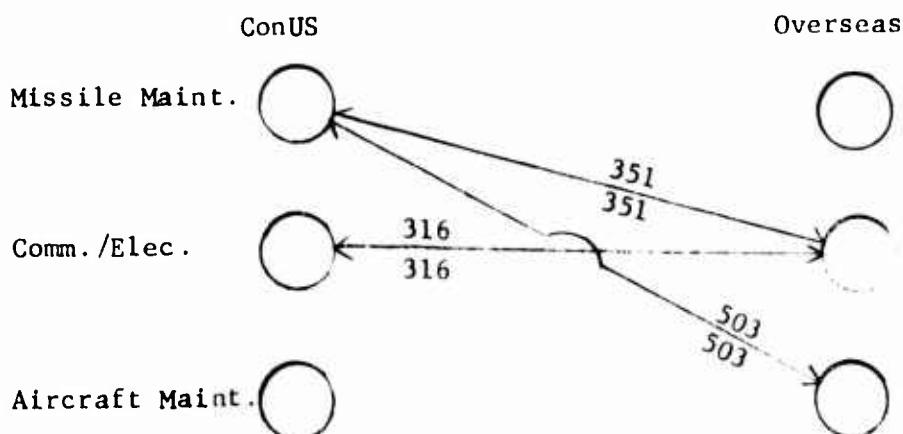


Fig. 3 -- Personnel Flow Pattern With  
Manpower Reduction of 4000 Overseas

The Rotation Base Model as it presently exists is more a refined estimating tool than a cross-training scheduling algorithm. This is due to the assumption of a system in equilibrium and the absence of dollar costs and training school capacities. We have found it somewhat difficult to obtain estimates of some of the parameters required both for this model and for its successor. Such things as the average ConUS tour length served in a given AFSC when that AFSC is in short supply, and what that AFSC is surplus, are not usually recorded, and the military personnel data files are still fairly cumbersome to access and manipulate for purposes of estimation.

We feel, however, that none of the problems we have mentioned--conceptual definitions of cost or value coefficients, computational feasibility, or data availability--are insurmountable. The insights provided by the exercise of this model in particular, and programming models in general, can be most valuable to the using agency and the decision-maker.

3. In the remainder of this paper we have briefly described some generalizations of the rotation situation which lead to familiar and pervasive manpower and personnel assignment problems. We have indicated how these problems may be formulated in a mathematical programming context, the type of information available, and the analyses which can be performed. More detailed treatment of these problems is given in the previously cited RAND Memorandum which is forthcoming.

A. A more realistic treatment of the rotation base problem must recognize accession and loss rates, dollar costs of training, cross-training, and rotation, capacity limitations of the training establishment, and the fact that instructor requirements may depend on student loads. One method of dealing with this problem is by framing it as a linear programming problem.

We might take as an objective function the total cost of enlisting, training, rotating, and separating the personnel force required to meet requirements in all tour areas. The rotation system must satisfy certain constraints. Movement must occur from each overseas tour area at a rate determined by the number of personnel in that area and the area tour length. Each period personnel are lost to the service from

every assignment class in every area at a known rate. Training schools leading to each assignment have a known capacity. We may assume that the number of instructors required in each training establishment is a linear function of the number of personnel being trained in that establishment. This problem can be formulated as choosing the rotation flows to minimize the total cost subject to the conditions listed--a linear programming problem. In addition to the obvious parametric analyses possible, the dual variables of the rotation flow constraints provide an indication of the tour areas which are particularly demanding in the rotational system.

B. Assume that personnel requirements by specialty and location are changing significantly each year. A steady state formulation may then not be appropriate. If, for instance, we know requirements by assignment class and tour area for each of five successive years, the problem may be viewed as a dynamic linear programming problem. We might use as a criterion the minimization of the cost of creating the manpower pool for the entire five years, where the decision variables are the cross-training flows and new accessions. The constraints affecting the system include the condition that requirements be met in all specialties in all periods, and the conditions relating the availability of specialists in one period to the number and type of specialists available the previous period. One might use this formulation to ask how much it would be worth in dollars to increase the retention rate in a particular specialty.

C. Suppose that requirements in the multi-period case are not known precisely, but that the probability distribution of the individual requirements can be estimated. Such problems can be dealt with in two ways.<sup>12</sup> A decision can be made for one period at a time, deferring later decisions until early requirement information is available. Solutions to sequential formulations of this type result in "strategies" rather than solutions--statements as to how the decision-maker should behave when more information becomes available to him.

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<sup>12</sup> Hadley, G., Nonlinear and Dynamic Programming, Addison-Wesley Reading, Massachusetts, 1964.

In planning for several years ahead one may have to solve the problem non-sequentially, or "once-and-for-all." In one situation of this type only the set of requirements will be random variables. In many industrial situations a penalty cost can be set on failure to meet requirements in any period, and this penalty cost can be entered into an appropriate objective function. If the resources provided are greater than the actual requirement in some period, there is once again a penalty. In the case of programming military personnel it is difficult to set a dollar cost or an effectiveness cost on personnel shortages, or on the effort required to reprogram resources into positions with shortages. An alternative method avoids the necessity of specifying penalty costs, but instead specifies some arbitrary probability with which each requirement constraint must be met. Solving the problem with a constraint involving an arbitrarily selected probability imputes a cost to that constraint in the optimal solution, which the decision-maker may not care to impute. It is therefore preferable to explicitly include penalty costs in the objective function. This can lead to a non-linear programming problem which has a separable concave objective function.

D. In an organization as large as the Air Force the allocation of manpower resources among major commands is a significant decision problem. It involves determining the distribution of contract service dollars, civil service authorizations, and military personnel authorizations in such a way that each command is able to perform its mission, and so that the overall effectiveness of the Air Force is maximized. Considered as a single large Air Force problem, the manpower relationships affecting the commands are fairly independent, while a few constraints affect all commands simultaneously. This type of structure is called a block diagonal problem.

There are several decomposition algorithms in existence which could be applied to this problem, but the most interesting is a recent algorithm by Zschau.<sup>13</sup> This algorithm appears to be the natural

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<sup>13</sup> Zschau, E., "A Primal Decomposition Algorithm for Linear Programming," Working Paper 91, Graduate School of Business, Stanford University, Stanford, California, January 1967.

computational analogue of bargaining in a decentralized structure, and seems sure to have a great deal of utility as a conceptual as well as a computational tool.

Using Zschau's approach, consider that the problem as seen by Headquarters USAF is maximization of overall effectiveness. The Air Force must utilize manpower resources which are of two types--those which are possessed solely by commands and those which are shared by all commands. An example of the former might be Depot Maintenance personnel, or R&D scientists, while Air Policemen are examples of the latter.

Each command faces the problem of using resources allocated to it in a way that maximizes its own effectiveness, while Headquarters must allocate common resources among the commands so that Air Force total effectiveness is maximized. Zschau shows conditions under which this problem is a concave programming problem and develops an algorithm to solve it, which seems to be a natural one for the analysis of allocation problems in large organizations.